Jurnal Akuntansi, Keuangan, dan Manajemen (JAKMAN)
ISSN 2716-0807, Vol 6, No 2, 2025, 399-408 https://doi.org/10.35912/jakman.v6i2.3803

The Application of Discrete Hilbert transform in Measuring Asset Return Risk in Investment Portfolios

Wahyuni Ekasasmita[1*], Khaera Tunnisa[2], Muh. Tri Aditya[3]
Institut Teknologi Bacharuddin Jusuf Habibie, Sulawesi Selatan, Indonesia[1,2,3]
wahyuni.ekasasmita@ith.ac.id[1*], khaeratunnisa@ith.ac.id[2], muhtriaditya47@gmail.com[3]

Article History
Received on 28 October 2024
1st Revision 12 November 2024
2nd Revision 2 December 2024
3rd Revision 15 December 2024
Accepted on 18 December 2024

Abstract

Purpose: This study aims to explore the application of the discrete Hilbert transform (DHT) in measuring asset return risk within a portfolio, emphasizing its potential for enhancing risk evaluation in the field of financial mathematics.

Methodology/approach: The research utilized a quantitative approach with data sourced from publicly available historical stock prices. Analysis was conducted using software MATLAB to implement the discrete Hilbert transform, which transforms time-series data into phase and amplitude components. The portfolio risk was calculated based on the transformed data, and the results were compared against traditional risk metrics such as variance and Value at Risk (VaR).

Results/findings: The findings indicate that the discrete Hilbert transform provides additional insights into portfolio risk by capturing frequency-domain characteristics of asset returns. It complements traditional measures, offering a novel perspective on risk analysis. Specifically, the discrete Hilbert transform was effective in identifying subtle changes in risk patterns that were not apparent in time-domain analyses alone.

Conclusion: This study investigates the application of the discrete Hilbert transform (DHT) in measuring the risk of asset returns within investment portfolios, specifically focusing on Indonesian stocks. The

Limitations: This study is limited by its focus on a small sample of stocks within a single financial market, which may restrict the generalizability of the findings. Additionally, the research does not account for macroeconomic factors that could influence asset returns.

Contribution: This study contributes to the field of financial risk management by introducing the discrete Hilbert transform as a supplementary tool for risk analysis. It offers practical implications for portfolio managers, actuaries, and financial analysts seeking innovative methods to enhance risk assessment and decision-making processes.

Keywords: *Asset Return Risk, Discrete Hilbert Transform, Investment Portfolio.*

How to Cite: Ekasasmita, W., Tunnisa, K., Aditya, M, T. (2025). The Application of Discrete Hilbert transform in Measuring Asset Return Risk in Investment Portfolios. *Jurnal Akuntansi, Keuangan, dan Manajemen*, 6(2), 399-408.

1. Introduction

In the real of finance, effective risk management is pivotal for optimizing investment portfolios and ensuring long-term stability (Kolari et al., 2024; MATEI & Buz, 2023; Qian, 2023). The measurement and analysis of risk have traditionally relied on various statistical and mathematical methods, each with its strengths and limitations. Among these, the discrete Hilbert transform (DHT) has emerged as a promising tool due to its capacity to capture and analyze complex financial data. Investment portfolios are subject to various risks, including market risk, credit risk, and liquidity risk (Manganelli & Engle, 2001; Miller, 2018; Seyedbarhagh et al., 2024). Market risk, for instance, involves fluctuations in asset prices that can impact the portfolio's value. Traditional methods of risk measurement, such as Value at Risk (VaR) and Conditional Value at Risk (CVaR), have been extensively used in finance (Angelovska, 2013; Brown, 2015; B. Li et al., 2024). However, these methods often rely on assumptions that may not hold true in all market conditions, leading to potential inaccuracies in risk assessment.

The Hilbert transform is a mathematical operation used to analyze signals and functions (Aliev & Səmədova, 2023; Y. Li & Yuan, 2019). In its discrete form, the discrete Hilbert transform (DHT) offers a way to extract phase and amplitude information from financial time series data (Ekasasmita et al., 2024). This method is particularly valuable in analyzing the cyclical nature of financial markets and identifying hidden patterns that traditional methods might overlook. The application of DHT in financial risk measurement is an evolving area of research. By applying DHT, analysts can gain a deeper understanding of the dynamics within asset returns and improve the accuracy of risk predictions. Recent studies have explored the use of DHT to enhance forecasting models and improve the robustness of risk management strategie (Uchiyama et al., 2019).

The novelty of applying DHT in the context of financial risk measurement lies in its ability to address some of the limitations inherent in traditional methods. For instance, while VaR provides a single threshold for risk, it does not account for the distribution's tails effectively. DHT, on the other hand, allows for a more nuanced analysis of the data, capturing both the amplitude and phase information, which can lead to more comprehensive risk assessments (Izza Anis Majidah et al., 2024). This paper aims to delve into the application of the discrete Hilbert transform in measuring the risk of asset returns within investment portfolios. By integrating this advanced mathematical tool into risk measurement frameworks, this research seeks to offer an improved methodology that enhances the accuracy and reliability of financial risk assessments.

Risk measurement in financial portfolios is a critical aspect of investment management. Traditional approaches, such as Value at Risk (VaR) and Conditional Value at Risk (Erickson, 2015) have been extensively used. VaR estimates the maximum potential loss over a specified period with a given confidence level. However, VaR has been criticized for not capturing extreme losses effectively, as it focuses on a single threshold and disregards tail risk. In CVaR, an extension of VaR, provides a more comprehensive view by considering the average loss beyond the VaR threshold (Seyedbarhagh et al., 2024). While these methods are useful, they are not without limitations, particularly in capturing the full spectrum of risk in volatile markets.

The Hilbert Transform is a mathematical operation that provides insights into the phase and amplitude of signals. It is widely used in signal processing and time-frequency analysis (Aliev & Səmədova, 2023). In financial applications, the Hilbert Transform can be applied to analyze the cyclical components of asset returns and identify underlying patterns not easily discernible through conventional methods. DHT is a variant of the Hilbert Transform adapted for discrete data sequences. It has gained attention for its ability to enhance the analysis of time series data, including financial time series (Ekasasmita et al., 2024). DHT provides valuable information about the instantaneous frequency and amplitude of financial returns, offering a different perspective compared to traditional risk measurement techniques. For example, studies have shown that DHT can improve the detection of cyclical patterns and anomalies in financial data, leading to more accurate risk assessments.

Recent research has explored the application of DHT in financial risk measurement. The DHT can decompose financial time series into components that reveal cyclical patterns and trends, which are crucial for understanding market behavior and assessing risk. Tehese components analyzed that investors can gain insights into potential risk factors that are not apparent in aggregate measures like VaR or CVaR. The application of DHT offers several advantages over traditional risk measurement methods. Unlike VaR, which provides a static measure of risk, DHT allows for a dynamic analysis of financial data, capturing both amplitude and phase information (Sugitomo & Maeta, 2020). This capability enables a more nuanced understanding of risk, particularly in volatile or rapidly changing market conditions. Furthermore, DHT's ability to analyze cyclical patterns can reveal insights into market trends that traditional methods might miss. Despite its advantages, the use of DHT in financial risk measurement also presents challenges. The complexity of the DHT computation and its sensitivity to noise in financial data can affect the accuracy of the results (Allen, 2012). Additionally, while DHT provides valuable insights, it should be used in conjunction with other risk measurement methods to ensure a comprehensive risk assessment.

2. Literature review

2.1 Risk in Investment and Portfolio Risk Measurement

Investment risk is a critical factor in portfolio management. One of the most common methods to measure risk is by using volatility, which refers to the standard deviation of asset returns. However, volatility does not always reflect the complete risk profile, especially in markets experiencing sharp changes or when price data are not normally distributed. Markowitz's theory introduced the portfolio theory that emphasizes the importance of diversification in reducing the total portfolio risk. However, this model is still limited in capturing risks that cannot be explained by standard volatility.

2.2 Hilbert Transform and Its Application in Finance

The Hilbert Transform is a mathematical tool used in signal analysis to convert signals into an analytic form that can represent both real and imaginary wave components. In finance, the Hilbert Transform can be used to analyze asset price movements and identify trends and finer patterns of volatility. One of the advantages of the Hilbert Transform compared to traditional techniques is its ability to separate high and low frequencies in price signals, allowing for more detailed analysis of market dynamics. The Discrete Hilbert Transform (DHT) is the discrete version of the Hilbert Transform, adapted for analyzing discrete data such as asset prices measured at specific time intervals. Several studies have shown that DHT can be used to identify and separate components of price signals that depend on long-term trends and short-term fluctuations (Karlina & Sanoyo, 2021). By utilizing DHT, investors and portfolio managers can gain deeper insights into hidden price patterns and more complex risks that are not captured by traditional methods.

2.3 Application of Discrete Hilbert Transform in Asset Risk Measurement

The application of DHT in measuring asset risk has become an interesting topic of research. Antonius and Tampubolon (2019) used DHT to analyze volatility and identify risks embedded in stock price movements. They found that DHT can better estimate risk components that are not detected by standard methods such as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. DHT allows for the separation between components that exhibit random fluctuations and those that represent more structured trends or patterns.

2.4 Comparison with Traditional Risk Measurement Methods

Traditional methods for measuring risk, such as volatility (standard deviation), Value at Risk (VaR), and Conditional Value at Risk (CVaR), often assume a normal distribution of asset returns. However, financial markets frequently operate outside this assumption, exhibiting fat tails and changing volatility. Therefore, these methods may not provide a complete picture of the risks faced by a portfolio. In contrast, the Discrete Hilbert Transform does not rely on specific distributional assumptions and is more flexible in capturing complex price movements. For example, DHT can identify components that may not be clearly visible through traditional analysis, such as the effects of market news or unexpected events that cause volatility spikes. With its ability to analyze both low- and high-frequency components, DHT provides a more precise tool for monitoring and measuring the risks embedded in investment portfolios.

2.5 Recent Studies on the Use of DHT in Portfolio Management

Several recent studies have also evaluated how the Discrete Hilbert Transform (DHT) can be applied in the context of portfolio management. Research by Abu (2024) indicates that by using DHT, portfolio managers can gain advantages in risk prediction and more accurate return forecasting. The findings suggest that DHT not only enhances the understanding of volatility but also provides better insights into tail risk, which is often a primary concern in portfolio management.

3. Methods

Mathematical Formulation of discrete Hilbert transform. The discrete Hilbert transform (DHT) of a discrete time series. The discrete Hilbert transform (DHT) provides a powerful tool for analyzing and processing discrete time-series data, particularly in financial markets. It offers a novel approach to capturing cyclical trends and hidden patterns in asset returns. The mathematical formulation of the DHT for a discrete time series can be expressed as:

$$H_D[r_k] = -isgn\left(k - \frac{T}{2}\right)\sum_{n=0}^{N-1} r_t e^{i\frac{2\pi t}{T}} \tag{1}$$

With sgn(.) signum function, r_t is the return of assets, and T denotes the period of the series, and k is the specific time point

In this study, the DHT was applied to analyze daily return data for three Indonesian stocks: PT Bank Rakyat Indonesia Tbk (BBRI.JK), PT Telkom Indonesia Tbk (TLKM.JK), and PT Unilever Indonesia Tbk (UNVR.JK). The data, obtained from Yahoo Finance, spans from January 1, 2019, to December 31, 2023, and includes daily return percentages.

Table 1. Data Description

Descriptive	BBRI	TLKM	UNVR
Count	1232	1231	1232
Mean	3560.31	3224.38	5401.12
Std	844.18	488.90	1619.32
Min	1654.78	2070.65	2942.96
25	3018.96	2859.06	3974.55
50	3382.76	3271.95	4649.02
75	4099.45	3626.47	6957.47
max	5433.86	4295.70	8247.25

Table 1 provides a summary of the data description, showcasing key statistics such as the mean, standard deviation, and range of prices for the selected stocks. These descriptive metrics set the foundation for deeper analyses. BBRI.JK Exhibits the highest variability, with a standard deviation (std) of 844.18, indicating greater price fluctuations compared to TLKM.JK and UNVR.JK. TLKM.JK Shows the lowest mean price and standard deviation, suggesting relative stability. UNVR.JK: Has the highest mean price (5401.12), reflecting its position as a high-value stock. These observations provide preliminary insights into the characteristics of each stock, guiding the selection of appropriate portfolio management strategies.

To evaluate the performance and risk profiles of different portfolio strategies, several mathematical metrics were utilized. Below, each metric is explained and contextualized within the framework of this study:

3.1 Annualized Return

$$AR = 1 + dR^{252} - 1 \tag{2}$$

With AR is annualized return which represent the compounded return over a year and dR represents the daily return. The annualized return captures the compounded return over a year, providing a standard measure for comparing the profitability of different strategies. EW and DHT strategies show modest positive annualized returns, indicating steady performance. The CW strategy exhibits a significant negative annualized return, reflecting poor asset allocation.

3.2 Annualized Volatility

$$AV = dV \times \sqrt{252} \quad (3)$$

With dV is daily volatility and annualized volatility measures the standard deviation of returns on an annualized basis. This metric translates daily price fluctuations into an annualized measure, offering a standardized way to assess market risk. The DHT strategy exhibits lower volatility compared to CW, underscoring its effectiveness in managing risk.

3.3 Skewness

$$Skewness = \frac{\sum_{i=1}^{n}(r_i - \mu)^3}{n \times \sigma^3} \quad (4)$$

Skewness indicates the asymmetry of the return distribution. Positive skewness implies a higher likelihood of extreme positive returns. Both EW and DHT exhibit positive skewness, signaling a preference for upside potential. The CW strategy's excessively high skewness highlights its vulnerability to rare, extreme events.

3.4 Kurtosis

$$Kurtosis = \frac{\sum_{i=1}^{n}(r_i - \mu)^4}{n \times \sigma^4} \quad (5)$$

Kurtosis indicates the "tailedness" of the distribution. A high kurtosis, especially in CW, suggests the presence of extreme outliers, making it riskier. DHT and EW demonstrate moderate kurtosis, aligning with their lower risk profiles. The CW strategy, with extreme kurtosis, reinforces its susceptibility to significant market shocks.

3.5 Historic Conditional Value at Risk (CVar)

$$CVar_{5\%} = \frac{\sum_{i=1}^{k} r_i}{k} \quad (6)$$

CVaR measures the average loss in the worst 5% of cases, providing a robust assessment of downside risk. DHT and EW strategies demonstrate relatively low CVaR values, showcasing their resilience in adverse conditions. The CW strategy’s high CVaR underscores its heightened risk exposure.

3.5 Sharpe Rasio

$$SR = \frac{AR - R_f}{AV} \quad (7)$$

R_f is the risk-free rate, often set to 0 in simplified cases. A negative Sharpe ratio for all strategies indicates that they perform worse than a risk-free asset (assuming $R_f = 0$). All strategies yield negative Sharpe ratios, indicating underperformance relative to risk-free assets.

3.6 Maximum Drawdown

$$MD = min\left(\frac{P_t - P_peak}{P_peak}\right) \tag{8}$$

Maximum drawdown represents the largest observed loss from peak to trough. The DHT strategy, despite modest drawdowns, demonstrates better recovery prospects compared to CW.

By analyzing the skewness, kurtosis, volatility, and drawdown metrics, we can observe how DHT provides insights into risk exposure and return distributions. Annualized Volatility and Historic Conditional Value at Risk (CVar) are two metrics that quantify the riskiness of the portfolio. Volatility indicates the level of uncertainty in the returns, while CVar shows the potential downside risk in extreme market conditions. Skewness and kurtosis are critical to understanding the distribution characteristics of asset returns. Skewness measures asymmetry, while kurtosis measures the fat tails, which represent extreme returns. The Sharpe Ratio measures the risk-adjusted return of the portfolios, and Max Drawdown assesses the maximum loss from the peak value of the portfolio. Both metrics are essential for evaluating the overall performance of an investment strategy. The comparison between DHT, EW (Equal Weight), and CW (Cap Weight) portfolios in the table highlights how the discrete Hilbert transform method (DHT) contrasts with traditional approaches in managing risk and returns.

4. Result and Discussion

Portfolio management strategies, including Equal Weight (EW), Capital Weighted (CW), and discrete Hilbert transform (DHT), exhibit diverse performance metrics based on annualized returns and volatility. These metrics are critical in evaluating risk-return dynamics and guiding investors toward more informed decision-making processes. The EW strategy yields an annualized return of approximately 0.0693%, a modest positive return indicating some level of portfolio stability. Similarly, the DHT strategy demonstrates a positive annualized return of 0.0125%, reflecting its potential in steady portfolio growth. However, the CW strategy starkly contrasts these findings, with a significant negative annualized return of -4.4783%. Such a substantial loss highlights the inherent risks in CW-based portfolio management, suggesting a potential misalignment with market dynamics or asset allocation inefficiencies.

Volatility, a measure of price fluctuation risk, varies greatly across the strategies. CW exhibits the highest annualized volatility of 312.36%, indicating extreme instability and heightened market exposure. On the other hand, EW and DHT strategies maintain much lower volatility levels at 5.01% and 4.81%, respectively, suggesting a better balance between risk and return. The lower volatility in these strategies underscores their suitability for risk-averse investors seeking more consistent performance outcomes. The skewness and kurtosis metrics reveal further insights into the return distributions of the portfolio strategies. Both EW and DHT exhibit positive skewness, suggesting that extreme positive returns are more frequent than extreme negative returns. This characteristic can be particularly attractive to investors seeking upside potential, even if overall returns remain modest. In contrast, the CW strategy demonstrates extraordinarily high skewness (18.02), reflecting its susceptibility to rare, extreme events—likely driven by poor diversification or concentration risk.

Kurtosis measures the "tailedness" of the return distributions. EW and DHT show relatively high kurtosis values (24.56 and 26.12, respectively), implying a propensity for infrequent but significant return deviations. CW, however, stands out with an exceedingly high kurtosis of 511.05, indicating extreme vulnerability to outliers. This reinforces the conclusion that the CW strategy is poorly equipped to handle market shocks, making it unsuitable for investors with low risk tolerance.

Figure 1. Stock Prices of Assets

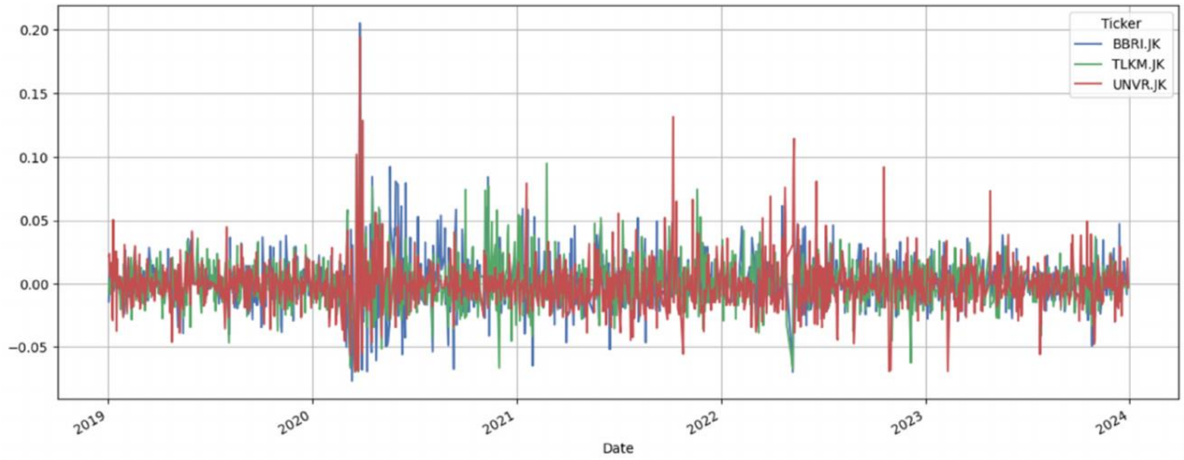

Figure 2. Return of Stock Prices

Figures 1 and 2 provide a visual representation of stock price movements and returns for BBRI.JK, TLKM.JK, and UNVR.JK from 2019 to 2024. These charts offer valuable insights into the volatility, correlation, and behavior of the selected assets over time.

The image displays a time series plot showing the daily returns of three different stocks: BBRI.JK (blue line), TLKM.JK (green line), and UNVR.JK (red line), spanning from 2019 to 2024. The graph provides a visual representation of the volatility and return behavior of these stocks over time. There are noticeable spikes in returns around the year 2020. This period likely corresponds to the market turbulence caused by the COVID-19 pandemic, where global markets experienced significant fluctuations. The spikes indicate days with unusually high returns (positive or negative), reflecting market instability. The three stocks exhibit some degree of correlation, as the lines often move together. However, there are instances where the movements diverge, indicating different reactions to market conditions. Most of the returns cluster around the zero line, suggesting that the majority of the daily returns are relatively small. This clustering around zero is typical in financial time series, where large price movements are less frequent. The returns for all three stocks generally fall within the range of -0.05 to 0.05, with some extreme outliers reaching beyond 0.2 or below -0.05. The range highlights the extent of price changes on any given day. After 2020, the volatility appears to decrease slightly, though the stocks still experience occasional spikes. This suggests some recovery and stabilization in the market post-pandemic, although not entirely returning to pre-pandemic levels.

Table 2. Result Analyze

Analyze	EW	CW	DHT

Annualized Return	0.000693	-0.044783	0.000125
Annualized Vol	0.0501175	3.123618	0.048070
Skewness	1.470736	18.025388	1.492360
Kurtosis	24.562954	511.059570	26.123780
Historic CVar (5 %)	0.031516	0.420060	0.051762
Sharpe Ratio	-0.568512	-0.023265	-0.505189
Max Drawdown	-0.403904	-6.481509	&- 0.416043

Source: Processing data Using Python

The historical Conditional Value at Risk (CVaR) at the 5% level quantifies potential losses during adverse market conditions. DHT showcases a CVaR of 5.17%, marginally higher than EW's 3.15%, indicating that both strategies provide a reasonable buffer against extreme market downturns. Conversely, CW demonstrates an alarming CVaR of 42.00%, reflecting substantial potential losses. Such findings highlight the necessity of incorporating robust risk management frameworks when utilizing CW-based strategies.

The Sharpe ratio, a risk-adjusted performance metric, further emphasizes the disparity among the strategies. Both EW and DHT yield negative Sharpe ratios (-0.5685 and -0.5052, respectively), indicating suboptimal risk-adjusted returns. CW, with a Sharpe ratio of -0.0233, fares slightly better but remains unsatisfactory. These results underline the importance of refining portfolio strategies to enhance risk-adjusted performance, especially under volatile market conditions. Maximum drawdown measures the largest peak-to-trough decline in portfolio value. The CW strategy suffers the most significant drawdown of -648.15%, emphasizing its inherent instability and inability to recover from market shocks effectively. EW and DHT strategies, with drawdowns of -40.39% and -41.60%, respectively, exhibit better resilience, though improvements in diversification and risk control mechanisms could further mitigate losses.

The stock price trends in Figure 1 reveal significant fluctuations, particularly during the COVID-19 pandemic in 2020. This period was characterized by unprecedented market volatility, as reflected in the steep declines and subsequent recoveries across the three stocks. Post-pandemic, the market shows signs of stabilization, though occasional price spikes suggest lingering uncertainties. Figure 2 highlights the daily returns of the stocks, with noticeable clustering around zero, a typical feature of financial time series. The presence of outliers, particularly during 2020, underscores the market's reaction to external shocks. The alignment of spikes across the three stocks indicates a degree of market-wide impact, while instances of divergence point to idiosyncratic factors affecting individual stocks.

The DHT strategy leverages the mathematical properties of the Hilbert Transform to decompose financial time series into amplitude and phase components. This decomposition enables a more nuanced understanding of market dynamics, including cyclical patterns and volatility trends. Amplitude analysis reveals the magnitude of price movements, offering insights into the intensity of market reactions. For instance, TLKM.JK exhibits higher amplitudes compared to BBRI.JK and UNVR.JK, suggesting greater sensitivity to market fluctuations. This information can inform asset allocation decisions, ensuring portfolios are better equipped to withstand market shocks. Phase analysis captures the cyclical nature of asset returns, highlighting periods of increased or decreased market activity. Such insights are invaluable for timing investment decisions and identifying potential turning points in market trends. By incorporating both amplitude and phase information, the DHT strategy provides a more comprehensive framework for portfolio management, enhancing both risk assessment and return optimization.

The CW strategy appears to be highly volatile and risky, with poor returns and a high potential for significant losses. EW and DHT are less risky but still show poor risk-adjusted returns. The phase component reveals cyclical patterns that could indicate periods of increased or decreased market risk. For instance, PT Telkom Indonesia Tbk (TLKM.JK) shows a higher mean amplitude and phase, suggesting greater volatility and more pronounced cyclical trends compared to other stocks. The DHT's ability to decompose financial time series into amplitude and phase components provides a more detailed view of market dynamics. This method could be particularly useful in developing enhanced risk management strategies that account for both volatility and cyclical trends.

The findings from this analysis offer several practical implications for investors. First, the stark differences in performance metrics among the strategies underscore the importance of aligning portfolio choices with individual risk preferences and investment objectives. For risk-averse investors, EW and DHT strategies emerge as more viable options, offering a balance between return potential and risk exposure. Second, the high volatility and poor risk-adjusted returns of the CW strategy highlight the need for caution when employing such approaches. Investors should consider enhancing diversification and incorporating advanced risk management techniques to mitigate the strategy's inherent vulnerabilities. Finally, the DHT strategy's ability to capture cyclical market patterns provides a unique advantage in dynamic market environments. By leveraging these insights, investors can develop more adaptive portfolio strategies, better equipped to navigate periods of heightened volatility.

5. Conclusion

This study investigates the application of the discrete Hilbert transform (DHT) in measuring the risk of asset returns within investment portfolios, specifically focusing on Indonesian stocks. The primary objective was to evaluate whether DHT provides a more nuanced and accurate risk measurement compared to traditional methods such as Value at Risk (VaR) and Conditional Value at Risk (CVaR). The application of DHT to the financial time series data allowed for the decomposition of asset returns into amplitude and phase components. The amplitude component, representing volatility, provided a more detailed view of risk compared to static measures like VaR and CVaR. For instance, the mean amplitude values revealed varying levels of volatility across different stocks, highlighting significant differences in risk exposure that traditional methods might not fully capture.

The comparative analysis showed that while VaR and CVaR are useful for estimating potential losses and understanding tail risk, they have limitations in capturing the full spectrum of risk. For example, VaR provides a threshold for potential loss but does not account for extreme outliers or the cyclical nature of financial markets. CVaR improves upon VaR by considering average losses beyond the threshold, yet it still lacks the granularity offered by DHT's amplitude and phase analysis. The integration of DHT into risk management frameworks offers several practical benefits. Financial analysts and portfolio managers can use DHT to gain deeper insights into the volatility and cyclical trends of asset returns, leadin1g to more informed investment decisions. This enhanced understanding of risk can contribute to better portfolio diversification strategies and more effective risk mitigation practices.

Despite its advantages, the study acknowledges certain limitations. The complexity of computing the DHT and its sensitivity to noise in financial data may affect the accuracy of the results. Additionally, while DHT provides valuable insights, it should be used in conjunction with other risk measurement methods to ensure a comprehensive assessment. Future research could explore the integration of DHT with advanced machine learning techniques to further refine risk predictions and enhance the robustness of risk management strategies. Additionally, extending the analysis to other asset classes and market conditions could provide a broader understanding of DHT's applicability and effectiveness.

Limitation

This study is limited by its focus on a small sample of stocks within a single financial market, which may restrict the generalizability of the findings. Additionally, the research does not account for macroeconomic factors that could influence asset returns.

Acknowledgement

This work was supported by Sains Department, Institut Teknologi Bacharuddin Jusuf Habibie, and LPPM-PM ITH (Research, Community Service, and Quality Assurance Institution).

Referensi

Abu, S. E. (2024). Audit committee characteristics and firm financial performance of quoted industrial goods firms in Nigeria. *International Journal of Financial, Accounting, and Management, 5*(4), 459-472. https://doi.org/10.35912/ijfam.v5i4.1718

Aliev, R., & Səmədova, L. (2023). Boundedness of the discrete Hilbert transform in discrete Hölder spaces. *Baku Mathematical Journal, 2*, 47–56. https://doi.org/10.32010/j.bmj.2023.04.

Allen, S. (2012). *Financial Risk Management: A Practitioner's Guide to Managing Market and Credit Risk* (2nd edition). Wiley.

Angelovska, J. (2013). Managing market risk with VaR (Value at Risk). *Journal of Contemporary Management Issues*, 18(2), 81-96.

Antonius, R., & Tampubolon, L. D. (2019). Analisis penghindaran pajak, beban pajak tangguhan, dan koneksi politik terhadap manajemen laba. *Jurnal Akuntansi, Keuangan dan Manajemen, 1*(1), 39-52.

Brown, A. (2015). *Financial Risk Management For Dummies* (1st ed). For Dummies.

Ekasasmita, W., Tunnisa, K., & Aditya, M. T. (2024). Forecasting Nonstationary Time Series Based on Dicrete Hilbert Transform. *Statistics, Optimization & Information Computing*. http://www.iapress.org/index.php/soic/article/view/2060.

Erickson, K. H. (2015). *Financial Risk Management: A Simple Introduction*.

Majidah, I., Rahim, A., & Bahri, M. (2024). Mean Variance Complex-Based Portfolio Optimization. *Statistics, Optimization & Information Computing*, *12*(5), 1382–1396. https://doi.org/10.19139/soic-2310-5070-2023.

Karlina, B., & Sanoyo, A. M. (2021). Pengaruh Cluster Emiten terhadap Return Saham JSX Berbasis Parameter Rasio Analisa Fundamental. *Jurnal Akuntansi, Keuangan dan Manajemen, 2*(4), 279-291.

Kolari, J., Liu, W., & Pynnonen, S. (2024). Net Long Portfolio Risk Analyses. *Springer Nature*. (169–189). https://doi.org/10.1007/978-3-031-48169-7_9.

Li, B., Zhou, R., He, Q., & Li, H. (2024). A generation expansion planning method considering conditional value-at-risk. *Journal of Physics: Conference Series*, *2782*, 012055. https://doi.org/10.1088/1742-6596/2782/1/012055.

Li, Y., & Yuan, Q. (2019). Phaseless inverse discrete Hilbert transform and determination of signals in shift-invariant space. *Mathematical Methods in the Applied Sciences*, *42(12), 4511-4527*. https://doi.org/10.1002/mma.5631.

Manganelli, S., & Engle, R. (2001). Value at Risk Models in Finance. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.356220.

Matei, V., & Buz, A. (2023). Behavioural finance perspective in theory of international portfolio investment. *Economics. Finances. Law, 4(1)*, 85–89. https://doi.org/10.37634/efp.2023.4.19.

Miller, M. B. (2018). *Quantitative Financial Risk Management* (1st edition). Wiley.

Qian, R. (2023). Management of Personal Finances and Investment Decisions. *Advances in Economics, Management and Political Sciences*, *64*, 202–212. https://doi.org/10.54254/2754-1169/64/20231532.

Seyedbarhagh, S., Laaksonen, H., & Karimi, M. (2024). *Assessing Economic Performance of an Energy Microgrid: A Conditional Value-at-Risk Optimization Approach* (pp. 227–233). https://doi.org/10.1007/978-3-031-59005-4_25.

Sugitomo, S., & Maeta, K. (2020). Quaternion Valued Risk Diversification. *Entropy*, *22*(4), 390. https://doi.org/10.3390/e22040390.

Uchiyama, Y., Kadoya, T., & Nakagawa, K. (2019). Complex Valued Risk Diversification. *Entropy*, *21*(2), 119. https://doi.org/10.3390/e21020119.